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Optimization of the Back-to-Front Suppression and Frequency Bandwidth Relationship for Three-Dimensional Acoustic Arrays

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ABSTRACT

For those three-dimensional arrays whose radiation-pattern function is the product of an end-fire-array-pattern function and a broadside-array-pattern function, a procedure has been developed for optimizing the frequency range vs the back-to-front-suppression relationship. That is, the amplitude coefficient distribution for the array is determined by a Tchebyscheff polynomial technique so that (a) the back-to-front suppression is maximum over a predetermined frequency range or (b) a specified back-to-front suppression is available over a maximum frequency range.

PROBLEM STATUS

This is an interim report on one phase of the problem; work is continuing.

AUTHORIZATION

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OPTIMIZATION OF THE BACK-TO-FRONT SUPPRESSION AND FREQUENCY BANDWIDTH RELATIONSHIP FOR THREE-DIMENSIONAL ACOUSTIC ARRAYS

INTRODUCTION

Faires (1) has shown that three-dimensional arrays whose elements are arranged in two or more planes parallel to the y-z plane may be used to produce unidirectional radiation patterns in the x-y plane (or any plane containing the x axis) over a wide range of frequencies in the interval $[0, 2f_0]$, where f_0 corresponds to quarter-wavelength spacing of the planes. As a measure of unidirectionality Faires used the bandwidth function $B(f)$ defined by

$$B(f) = \frac{R(f,\pi)}{R(f,0)} \tag{1}$$

where $R(f,\gamma)$ is the far-field pattern function of the array for the x-y plane, f is the driving frequency, and γ is the angle measured from the x axis to the field point.

For those arrays whose elements are arranged in any number of planes oriented parallel to the y-z plane and whose radiation pattern in the x-y plane is the product of an end-fire-array-pattern function and a broadside-array-pattern function this report describes a technique for optimizing the bandwidth function. That is, the amplitude distribution for the end-fire dimension is determined so that (a) if $r > 1$ is fixed, then $|B(f)| \leq 1/r < 1$ over a maximum range of frequencies in the interval $[0, 2f_0]$ or (b) if $0 < f_1 \leq f \leq f_2 < 2f_0$, where $f_2 = 2f_0 - f_1$, then $1/r$ is the smallest number such that $|B(f)| \leq 1/r$.

THREE-PLANAR ARRAY AS AN EXAMPLE

Before proceeding to the analysis of arrays containing n planes it is helpful to demonstrate the interdependence of frequency bandwidth, unidirectional radiation, and radiating-surface-velocity distribution for a specific array. For example, consider the 36-element, three-planar array in Fig. 1.

If the radiating-surface-velocity distribution of the array is given by

$$\text{plane 1:}\quad v_1 = v_0\, e^{i\omega t}$$

$$\text{plane 2:}\quad v_2 = a\, v_0\, e^{i(\omega t + \psi_0)}$$

$$\text{plane 3:}\quad v_3 = v_0\, e^{i(\omega t + 2\psi_0)}$$

then the radiation-pattern function for the array is given (1) by

$$R(f,\gamma) = \frac{a + 2\cos(\psi - \psi_0)}{(2 + a)}\ \frac{(\cos\varphi + \cos 3\varphi + \cos 5\varphi)}{3} \tag{2}$$

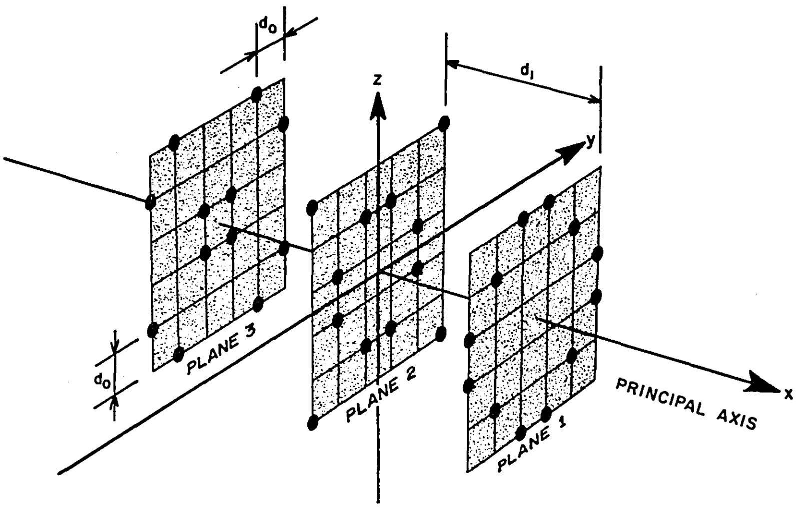


Fig. 1 - A typical 36-element, three-planar array (shown expanded along the principal axis)

where

$$\varphi = \pi \frac{d_0}{\lambda} \sin \gamma = \pi \frac{d_0}{c} (f \sin \gamma) ,$$

$$\psi = 2\pi \frac{d_1}{\lambda} \cos \gamma = 2\pi \frac{d_1}{c} (f \cos \gamma) = \frac{\pi}{2} \frac{f}{f_0} \cos \gamma ,$$

ψ_0 is a phase delay function, and f_0 is the frequency corresponding to quarter-wavelength spacing of the planes of the array. Note that $R(f,\gamma)$ is the product of an end-fire-array-pattern function and a broadside-array-pattern function.

The bandwidth function is

$$B(f) = \frac{a + 2 \cos \left(2\pi \frac{d_1}{c} f + \psi_0\right)}{a + 2 \cos \left(2\pi \frac{d_1}{c} f - \psi_0\right)} . \tag{3}$$

If

$$\psi_0 = 2\pi \frac{d_1}{\lambda} = 2\pi \frac{d_1}{c} f ,$$

then $R(f,0) = a + 2$ and

$$B(f) = \frac{1}{a + 2} \left(a + 2 \cos \pi \frac{f}{f_0}\right) . \tag{4}$$

If $0 < a < 2$, then the unidirectional properties of the array are given by

$$B(0) = B(2f_0) = 1$$

$$B(f_0) = \frac{a-2}{a+2}$$

$$|B(f)| \le \left|\frac{a-2}{a+2}\right| < 1\,, \quad \text{for} \quad f_1 \le f \le f_2$$

where

$$f_1 = f_0 \frac{\arccos\,(1-a)}{\pi} \tag{5}$$

$$f_2 = 2f_0 - f_1\,. \tag{6}$$

The bandwidth function for the 36-element, three-planar array is given in Fig. 2 for six different values of a. The limiting values $a = 0$ and $a = 2$ are included.

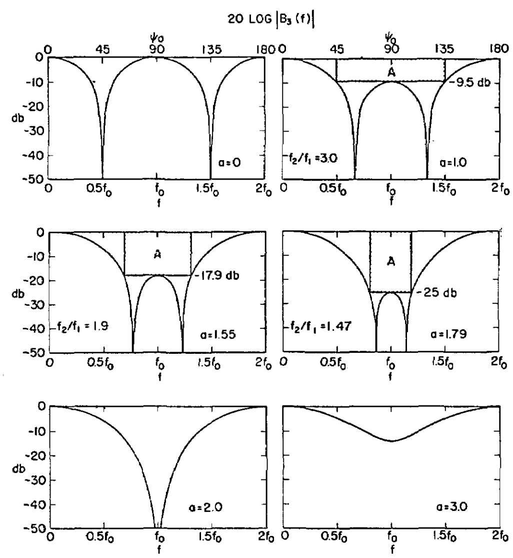


Fig. 2 - The bandwidth functions for the three-planar array of Fig. 1 for different magnitudes of a, the relative magnitude of the radiating surface velocity of the elements in the center plane

ANALYSIS OF n-PLANAR ARRAYS

The ensuing analysis covers those three-dimensional, n-planar arrays whose radiation pattern in the x-y plane is given by

$$R_n(f,\gamma) = F_n(f,\gamma)\; G_m(f,\gamma)\; H(f,\gamma)\,. \tag{7}$$

where m is the number of element positions in each plane relative to the y axis.

For an even number (n) of planes

$$F_n(f,\gamma) = \sum_{j=1}^{n/2} A_j \cos\left[\left(\frac{2j-1}{2}\right)\left(2\pi \frac{d_1}{c} f \cos\gamma - \alpha\right)\right] \tag{8}$$

and for any odd number (n) of planes

$$F_n(f,\gamma) = \sum_{j=0}^{\frac{n-1}{2}} A_j \cos\left[j\left(2\pi \frac{d_1}{c} f \cos\gamma - \alpha\right)\right] \tag{9}$$

where γ is the angle measured from the x axis, f is the driving frequency, c is the sound velocity in the medium (water), and α is a phase difference between elements in adjacent planes.

When m is even,

$$G_m(f,\gamma) = \sum_{k=1}^{m/2} B_k \cos\left[\left(\frac{2k-1}{2}\right)\pi \frac{d_0}{c} f \sin\gamma\right] . \tag{10}$$

A similar expression exists for $G_m(f,\gamma)$ whenever m is odd; however, it is assumed that m is even.

The radiation pattern of the individual elements is given by $H(f,\gamma)$. It is assumed that

$$H(f,\gamma) = H(f,-\gamma) = H(f,\pi-\gamma) . \tag{11}$$

The simplest array having $R(f,\gamma)$ as its radiation-pattern function is the rectangular, two-dimensional array which contains $n \cdot m$ elements and lies (along the principal axis) in the x-y plane. The locations of the elements in the x-y plane are given by

$$y_k = \frac{k}{|k|}\left(\frac{2|k|-1}{2}\right) d_0 \tag{12}$$

where

$$-\frac{m}{2} \le k \le \frac{m}{2}, \quad k \neq 0 .$$

If n is even, then

$$x_j = \frac{j}{|j|}\left(\frac{2|j|-1}{2}\right) d_1 \tag{13}$$

where

$$-\frac{n}{2} \le j \le \frac{n}{2}, \quad j \neq 0$$

of if n is odd, then

$$x_j = j d_1 \tag{14}$$

where

$$-\frac{(n-1)}{2} \leq j \leq \frac{n-1}{2}.$$

The radiating-surface-velocity of the projector located at (x_j, y_k) is v_{jk}. If n is even, then

$$v_{jk} = A_j B_k e^{i\{\omega t+(k/|k|)[(2|k|-1)/2]\psi_0\}}. \tag{15}$$

If n is odd, then

$$v_{jk} = A_j B_k e^{i(\omega t+j\psi_0)} \tag{16}$$

except that the radiating surface velocity of the elements in the center row $(j = 0)$ is given by

$$v_{0k} = 2A_0 B_k e^{i\omega t}. \tag{17}$$

Starting with this two-dimensional array as a reference, it is possible to construct any number of different three-dimensional arrays so that their radiation pattern in the x-y plane is given by Eq. (7). The three-dimensional array is constructed in the following manner. At each of the $n \cdot m$ points (x_j, y_k) define the line parallel to the z axis as the (j,k)th riser. If on each of the risers there are q elements and if on the (j,k)th riser all q elements have the same radiating surface velocity, then the radiation pattern for the three-dimensional array containing $n \cdot m \cdot q$ elements is also given (2) by Eq. (7).

It is not the intent of this report to promote any specific array design; however, the symmetric cyclic arrays discussed by Faires (1) are obtained by setting $m = 2n$ and $q = 2$.

For arrays containing $2m + 1$ planes the bandwidth function is given by

$$B_{2m+1}(f) = \frac{\left|\sum_{j=0}^{m} A_j \cos j\left(\frac{\pi}{2}\frac{f}{f_0} + \alpha\right)\right|}{\left|\sum_{j=0}^{m} A_j \cos j\left(\frac{\pi}{2}\frac{f}{f_0} - \alpha\right)\right|} \tag{18}$$

and for arrays containing $2m$ planes the bandwidth function is given by

$$B_{2m}(f) = \frac{\left|\sum_{j=0}^{m-1} A_{j+1} \cos \frac{2j+1}{2}\left(\frac{\pi}{2}\frac{f}{f_0} + \alpha\right)\right|}{\left|\sum_{j=0}^{m-1} A_{j+1} \cos \frac{2j+1}{2}\left(\frac{\pi}{2}\frac{f}{f_0} - \alpha\right)\right|}. \tag{19}$$

The equation

$$\alpha = \psi_0 = \frac{\pi}{2}\frac{f}{f_0} \tag{20}$$

is a requirement for a constant time delay between the radiating-surface velocities of the elements in any two adjacent planes (parallel to the y-z plane) such that the pressure waves from all projector elements add in phase at all frequencies in the far field for $\gamma = 0°$. Unless it is explicitly stated otherwise herein, α is given by Eq. (20).

The bandwidth functions become

$$B_{2m+1}(f) = \frac{\left| \sum_{j=0}^{m} A_j \cos 2j \frac{\pi}{2} \frac{f}{f_0} \right|}{\sum_{j=0}^{m} A_j} \tag{21}$$

$$B_{2m}(f) = \frac{\left| \sum_{j=0}^{m-1} A_{j+1} \cos \left[(2j+1) \frac{\pi}{2} \frac{f}{f_0} \right] \right|}{\sum_{j=0}^{m-1} A_{j+1}}. \tag{22}$$

Equations (21) and (22) may be written in the form

$$\overline{B}_{2m+1}(x) = \frac{\sum_{j=0}^{m} C_j x^{2j}}{\sum_{j=0}^{m} A_j} \tag{23}$$

$$\overline{B}_{2m}(x) = \frac{\sum_{j=0}^{m-1} C_{j+1} x^{2j+1}}{\sum_{j=0}^{m-1} A_{j+1}} \tag{24}$$

where $x = \cos [(\pi/2)(f/f_0)]$ and the C_j's are linear functions of the A_j's. The bar over $B_{n+1}(f)$ (i.e., $\overline{B}_{n+1}(x)$) means that $B_{n+1}(f)$ is considered as a polynomial of degree n in x.

For example, the bandwidth function, $B_5(f)$, for a five-planar array

$$B_5(f) = \frac{\left| A_0 + A_1 \cos 2\left(\frac{\pi}{2} \frac{f}{f_0}\right) + A_2 \cos 4\left(\frac{\pi}{2} \frac{f}{f_0}\right) \right|}{A_0 + A_1 + A_2} \tag{25}$$

becomes

$$\overline{B}_5(x) = \frac{C_0 + C_1 x^2 + C_2 x^4}{A_0 + A_1 + A_2} \tag{26}$$

where

$$C_0 = 8A_2$$

$$C_1 = 2A_1 - 8A_2$$

$$C_2 = A_0 - A_1 + A_2 .$$

Equations (23) and (24) satisfy

$$\overline{B}_{n+1}(1) = 1 \tag{27}$$

and

$$\overline{B}_{n+1}(-x) = (-1)^n \, B_{n+1}(x) . \tag{28}$$

The bandwidth function $B_{n+1}(f)$ is optimized by equating $\overline{B}_{n+1}(x)$ to the normalized Tchebyscheff polynomial $Q_n(x)$ which is of degree n, has real coefficients, has only even or odd powers of x depending on whether n is even or odd, and is defined by

$$Q_n(x) = \frac{T_n(z)}{T_n(z_0)} = \frac{T_n(z_0 x)}{T_n(z_0)} \tag{29}$$

where $T_n(z)$ is the nonnormalized Tchebyscheff polynomial* of degree n. $Q_n(x)$ satisfies Eqs. (27) and (28), and in addition

$$|Q_n(x)| \leq \frac{1}{T_n(z_0)} < 1 \tag{31}$$

if $- 1/z_0 \leq x \leq 1/z_0 < 1$. If $\overline{B}_{n+1}(x) = Q_n(x)$, then

$$|B_{n+1}(f)| \leq \frac{1}{T_n(z_0)} \tag{32}$$

if $0 < f_1 \leq f \leq f_2 < 2f_0$, where

$$f_1 = \frac{2f_0}{\pi} \arccos\left(\frac{1}{z_0}\right) \tag{33}$$

and

$$f_2 = \frac{2f_0}{\pi} \arccos\left(-\frac{1}{z_0}\right) . \tag{34}$$

It follows from Eqs. (33) and (34) that

$$f_2 = 2f_0 - f_1 . \tag{35}$$

It has been shown (3), in the general case, whenever

$$\overline{B}_{n+1}(x) = Q_n(x)$$

*The nonnormalized Tchebyscheff polynomials are given in the Appendix.

the coefficients A_j (of $B_{n+1}(f)$) can be obtained as functions of z_0 by first solving for A_j, then A_{j-1}, then A_{j-2}, etc. For example, the equation

$$\overline{B}_5(x) = Q_4(x)$$

becomes

$$\frac{C_0 + C_1 x^2 + C_2 x^4}{A_0 + A_1 + A_2} = \frac{1 - 8z_0^2 x^2 + 8z_0^4 x^4}{1 - 8z_0^2 + 8z_0^4}$$

where C_1, C_2, C_3 are defined by Eq. (26). Hence

$$A_2 = z_0^4$$

$$A_1 = 4z_0^4 - 4z_0^2$$

$$A_0 = 3z_0^4 - 4z_0^2 + 1 .$$

The amplitude distributions which optimize the bandwidth functions for arrays containing three, four, and five planes are plotted in Fig. 3 as a function of $20 \log (1/r)$.

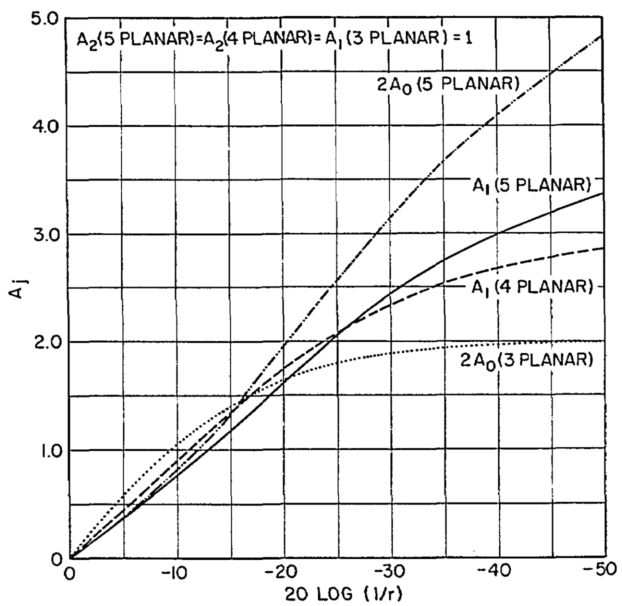


Fig. 3 - The relative values of the coefficients of the optimum amplitude distribution as a function of back-to-front suppression for arrays of three, four, and five planes

If it is desired that $|\bar{B}_{n+1}(x)| \leq 1/r$ in an interval centered on f_0, then set

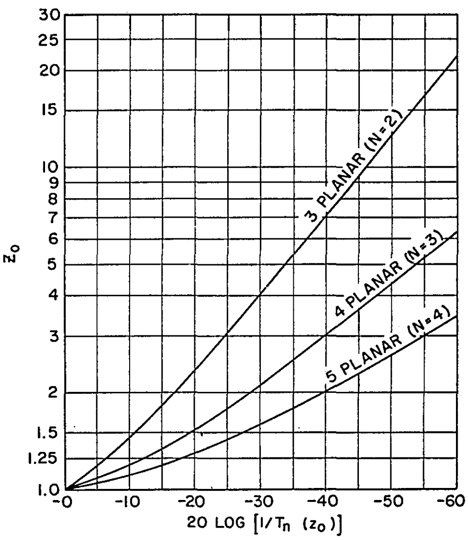
$$T_n(z_0) = r .$$

The quantities z_0, n, and r are related (5) by

$$z_0 = \frac{1}{2}\left[\left(r + \sqrt{r^2 - 1}\right)^{1/n} + \left(r - \sqrt{r^2 - 1}\right)^{1/n}\right]. \tag{36}$$

Equation (36) is plotted in Fig. 4 for $n = 2$, 3, and 4.

Fig. 4 - The scale contraction z_0 as a function of $1/T_n(z_0)$ for arrays of three, four, and five planes



The amplitude distribution (the set of A_j's) is determined as a function of z_0, and the frequency range $[f_1, f_2]$ is given by Eqs. (33) and (34). The range $[f_1, f_2]$ is the maximum frequency range over which $|B_{n+1}(f)| \leq 1/r$. If the frequency range is specified (i.e., $[f_1, f_2]$, where $f_2 = 2f_0 - f_1$), then the maximum back-to-front suppression in $[f_1, f_2]$ is

$$|B_{n+1}(f)| \leq \frac{1}{T_n(z_0)} \tag{37}$$

where $f_1 \leq f \leq f_2$ and

$$z_0 = \frac{1}{\cos \frac{\pi}{2} \frac{f}{f_0}} = \frac{1}{-\cos \frac{\pi}{2} \frac{f_2}{f_0}} . \tag{38}$$

The amplitude distribution is determined in the manner just described. The proof of these last two assertions follows from the theorem below.

Theorem: Of all polynomials of degree n with real coefficients such that $P(1) = 1$ and $P(-x) = (-1)^n P(x)$ the normalized Tchebycheff polynomial

$$Q_n(x) = \frac{T_n(z_0 x)}{T_n(z_0)}$$

either (a) deviates least from the abscissa in the range of x from $-1/z_0$ to $1/z_0$ inclusively or (b) is less than or equal to $1/T_n(z_0)$ over a maximum range of x in the interval $[-1, 1]$.

Proof: Only the proof for (a) is given. The proof of (b) follows easily from the proof of (a). Assume that a polynomial $S_n(x)$ of degree n with real coefficients which satisfies the conditions $S_n(1) = 1$, $S_n(-x) = (-1)^n S(x)$, and $|S_n(x)| \leq 1/S < 1/r = 1/T_n(z_0)$ for $-1/z_0 \leq x \leq 1/z_0$. The nonnormalized Tchebyscheff polynomial $T_n(z_0 x) = T_n(z)$ touches the lines $y = \pm 1$ at $(n+1)$ points in the interval $-1 \leq z \leq 1$. Consequently, $Q_n(x)$ touches the lines $y = \pm 1/T_n(z_0)$ at $(n+1)$ points in the interval $-1/z_0 \leq x \leq 1/z_0$. Denote these $(n+1)$ points by

$$x_1 = \frac{1}{z_0}, \; x_2, \; x_3, \; \ldots, \; x_{n+1} = -\frac{1}{z_0}.$$

The difference polynomial,

$$D(x) = Q_n(x) - S_n(x), \tag{39}$$

is evaluated at $x_1, x_2, \ldots, x_{n+1}$:

$$D(1) = 0$$

$$D(x_1) > 0$$

$$D(x_2) < 0$$

$$D(x_3) > 0$$

$$D(x_n) \begin{cases} < 0 \text{ if } n \text{ is even} \\ > 0 \text{ if } n \text{ is odd} \end{cases}$$

$$D(x_{n+1}) \begin{cases} > 0 \text{ if } n \text{ is even} \\ < 0 \text{ if } n \text{ is odd} \end{cases}$$

$$D(-1) = 0.$$

$D(x)$ changes sign n times in the interval $[x_1, x_{n+1}]$, so that $D(x)$ has n roots in the same interval. $D(x)$ has two additional roots at ± 1. This means $D(x)$ is of degree $(n+2)$, which is impossible. Hence $S_n(x)$ has a greater deviation from the abscissia for $-1/z_0 \leq x \leq 1/z_0$ than does $Q_n(x)$. Q.E.D.

The ratio

$$\frac{f_2}{f_1} = \frac{\arccos(-1/z_0)}{\arccos(+1/z_0)} \tag{40}$$

is a measure of the frequency range over which the maximum back-to-front suppression is obtained (see Eqs. (33) and (34), which define f_1 and f_2). It follows from Eq. (36) that f_2/f_1 is a function of both n and r, where $(n+1)$ is the number of planes in the array and $1/r$ is the maximum value of $|B_{n+1}(f_0)|$ between the frequencies f_1 and f_2. The ratio f_2/f_1 is plotted as a function of $(n+1)$ and $20 \log(1/r)$ in Fig. 5.

A figure of merit for multiplanar array operation is the relative area of the inscribed rectangle in the graphs of the bandwidth function (for example, see Fig. 2). The area of the inscribed area is plotted in Fig. 6 for arrays containing two, three, four, and five planes.

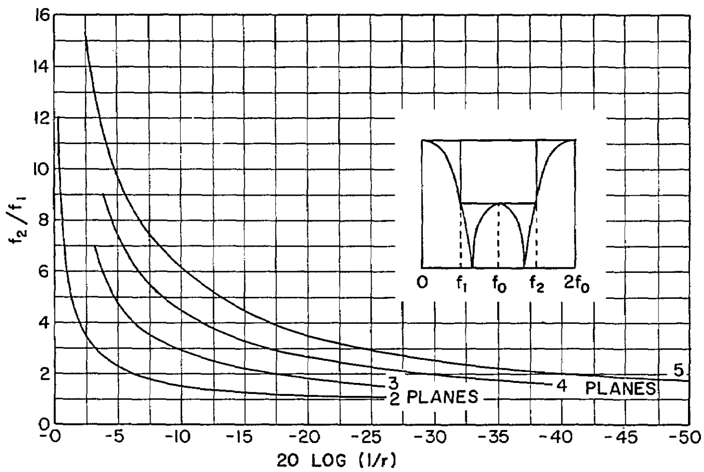


Fig. 5 - The ratio f_2/f_1 as a function of back-to-front suppression for arrays of two, three, four, and five planes, where f_2 is the maximum and f_1 is the minimum frequency at which the desired back-to-front suppression is obtained

Fig. 6 - The area of the inscribed rectangle is a figure of merit for the multiplanar arrays. Here it is plotted against the back-to-front suppression for arrays containing two, three, four, and five planes.

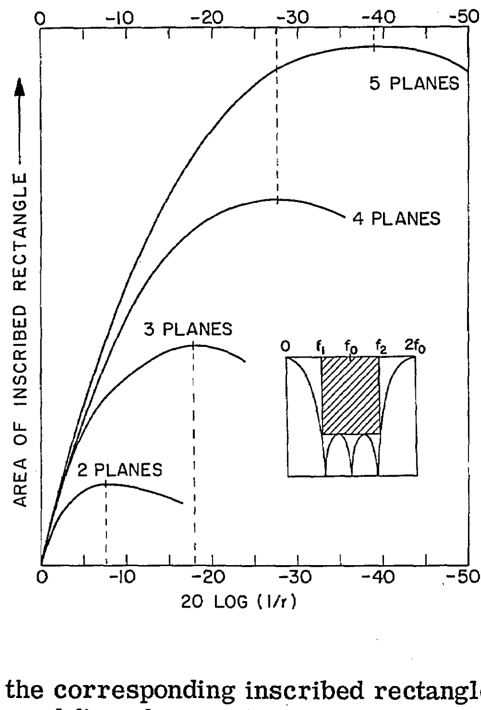


Figure 7 gives the bandwidth function and the corresponding inscribed rectangles for multiplanar arrays containing two, three, four, and five planes. In each case, the back-to-front suppression corresponds to the maximum area of the inscribed rectangle in Fig. 6. In each of the four cases in Fig. 7, the maximum area of the inscribed rectangle corresponds approximately to $f_2/f_1 = 2$.

It has been shown that the amplitude distribution (see Fig. 3) $2A_0 = 2.67$, $A_1 = 2.11$, and $A_2 = 1.00$ and $\alpha = \psi_0 = (\pi/2)(f/f_0)$ gives -26-db back-to-front suppression over a maximum frequency range. The following example illustrates the use of Fig. 5. When

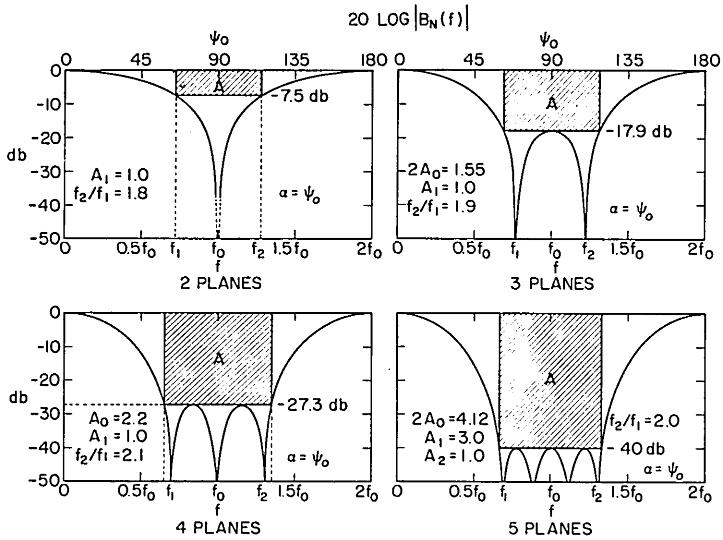


Fig. 7 - Bandwidth functions for two, three, four, and five planes. The optimum amplitude distributions shown give the rectangles of maximum area in Fig. 6.

$f_2/f_1 = 2.8$ is placed in Eqs. (33), (34), and (35), we see that $f_1 = 0.52\ f_0$ and $f_2 = 1.48\ f_0$ or that $20 \log|B(f)| = -26$ whenever $0.52\ f_0 \leq f \leq 1.48\ f_0$. Figure 8 gives the F-factor radiation patterns for the amplitude distribution which gives -26 db back-to-front suppression and $\alpha = \psi_0$ over a maximum range of frequencies for the five-planar array.

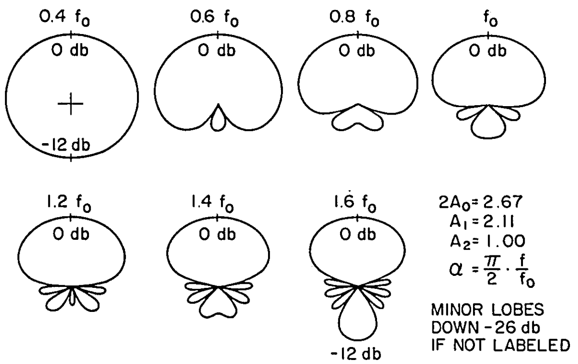


Fig. 8 - F-factor beam patterns for a five-planar array. The amplitude distribution given is optimum in that the back-to-front suppression is -26 db over a maximum frequency range (from 0.52 f_0 to 1.48 f_0). The phase difference α represents a constant time delay.

Figure 9 gives the F-factor radiation for the same array and operating conditions of Fig. 8 except that $\alpha = \pi/2$. Although the F-factor patterns in Fig. 9 seem, on the whole, more desirable than those in Fig. 8, it is apparent that -26-db back-to-front suppression is achieved only at the center frequency f_0. The array designer would of course use a sufficiently large number of elements in each plane to reduce the width of the main beam (see Fig. 10). Thus the width of the main beam of the F-factor pattern in Fig. 8 is of little concern here.

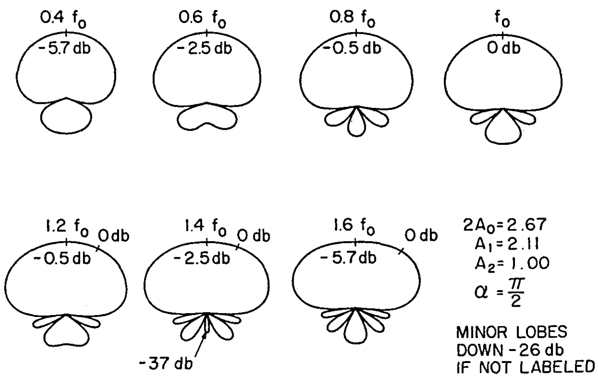


Fig. 9 - Same as Fig. 8 except that $\alpha = \pi/2$ represents a constant phase difference

The beam pattern for the multiplanar array is given by Eq. (7). Figure 10 illustrates the pattern multiplication of a typical F-factor and G-factor pattern. It is assumed that

$$H(f,\gamma) = 1 .$$

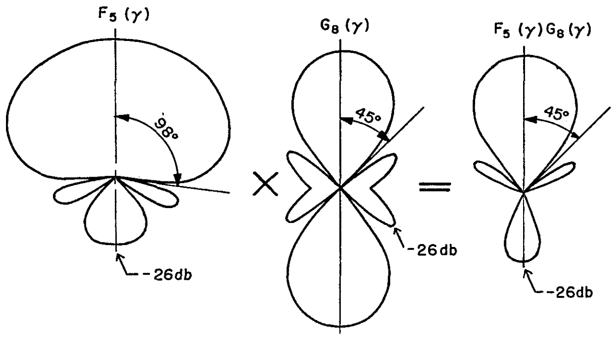


Fig. 10 - Pattern multiplication of a five-element F-factor pattern and an eight-element G-factor pattern. The F-factor amplitude distribution ($2A_0$ = 2.67, A_1 = 2.11, A_2 = 1.00) is optimum in that the 26-db back-to-front suppression is obtained over a maximum frequency range (see Fig. 3). The G-factor amplitude distribution (B_0 = 2.9, B_1 = 2.4, B_2 = 1.6, B_3 = 1.0) is optimum (3) in that the main beam has the minimum width for a -26-db side-lobe level and $d_0/\lambda = 1/4$.

It is possible to obtain the F-factor beam pattern from the bandwidth function. For example, the F-factor pattern for an array containing $2m+1$ planes at the frequency f_p is given by

$$\frac{F_{2m+1}(f_p, \gamma)}{F_{2m+1}(f_p, 0)} = \frac{\sum_{j=0}^{m} A_j \cos j\left[\frac{\pi}{2}\frac{f_p}{f_0}(\cos\gamma - 1)\right]}{\sum_{j=0}^{m} A_j} \tag{41}$$

where $\alpha = \psi_0 = (\pi/2)(f_2/f_0)$. Equation (41) is equated to Eq. (21). Since the arguments of the two functions must be equal, it follows that

$$\frac{\pi}{2}\frac{f_p}{f_0}(\cos\gamma - 1) = -\left(2\frac{\pi}{2}\frac{f}{f_0}\right) \tag{42}$$

where $0 \leq \gamma \leq \pi$ and $0 \leq f \leq f_p$. Finally,

$$\gamma = \arccos\left(1 - \frac{2f}{f_p}\right). \tag{43}$$

This procedure is illustrated in Fig. 11 for the five-planar array and $f = f_2$.

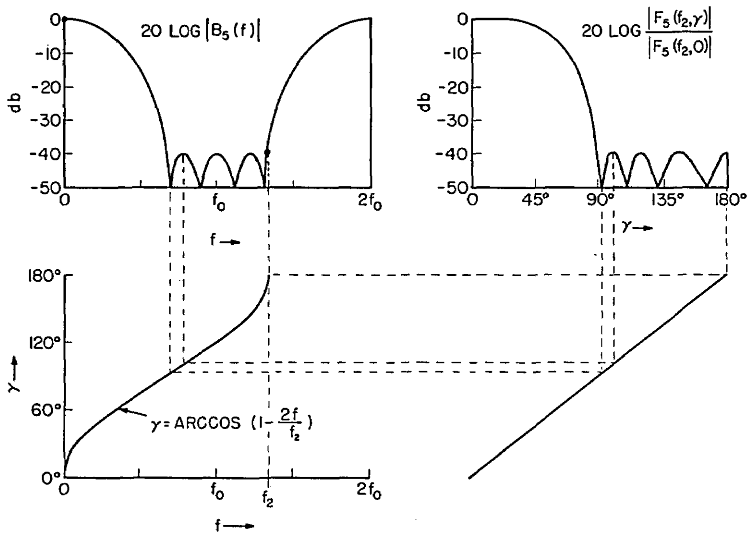


Fig. 11 - Graphical procedure for obtaining the F-factor beam pattern from the bandwidth function for a five-planar array. In the case illustrated f_p corresponds to f_2 (the maximum frequency at which a 40-db back-to-front suppression will exist).

The F-factor radiation patterns are given in Fig. 12 for the five-planar array, the amplitude distribution $2A_0 = 7.88$, $A_1 = -5.88$, and $A_2 = 2.16$, and the time delay $\alpha = 0.117\ f/f_0$, which gives the optimum F-factor pattern at a single frequency (4).

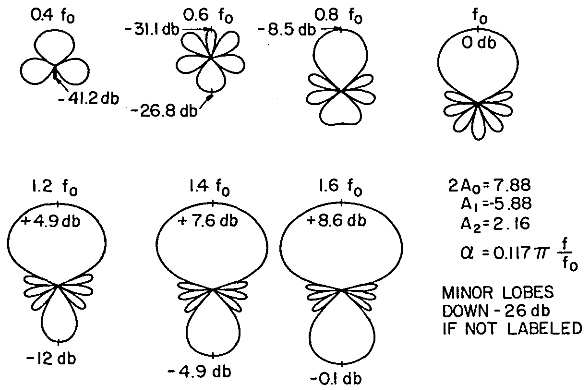


Fig. 12 - F-factor or bandwidth-function beam patterns for the amplitude distribution which gives the optimum end-fire beam pattern at the single frequency f_0 (3). The phase difference α represents a constant time delay.

In summary, the bandwidth function $B_{n+1}(f)$ may be optimized by equating it to the normalized Tchebyscheff polynomial $Q_n(z_0 \cos \psi_0)$. That is, the amplitude distribution of $B_{n+1}(f)$ (i.e., the set of A_j's) is determined so that (a) the range of frequencies will be maximum over which $|B_{n+1}(f)| \leq K < 1$ (K arbitrary) or (b) for a given range of frequencies K will be the smallest number for which $|B_{n+1}(f)| \leq K < 1$.

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APPENDIX

NONNORMALIZED TCHEBYSCHEFF POLYNOMIALS

The nonnormalized Tchebyscheff polynomials are given* by

$$T_n(z) = \cos(n \arccos z) \quad \text{for } -1 \leq z \leq 1$$

written in polynomial form where $-\infty < z < +\infty$

$$T_0(z) = 1$$

$$T_1(z) = z$$

$$T_2(z) = 2z^2 - 1$$

$$T_3(z) = 4z^3 - 3z$$

$$T_4(z) = 8z^4 - 8z^2 + 1 .$$

They satisfy the recursion formula

$$T_{n+1}(z) = 2z\, T_n(z) - T_{n-1}(z) \quad (n \geq 1) .$$

* * *

*C.L. Dolph, Proc. IRE 34:335-348 (June 1946).

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For those three-dimensional arrays whose radiation-pattern function is the product of an end-fire-array-pattern function and a broadside-array-pattern function, a procedure has been developed for optimizing the frequency range vs the back-to-front-suppression relationship. That is, the amplitude coefficient distribution for the array is determined by a Tchebyscheff polynomial technique so that (a) the back-to-front suppression is maximum over a predetermined frequency range or (b) a specified back-to-front suppression is available over a maximum frequency range.

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